Mathematical Logic Honours / 2008

Assignment 4

February 28, 2008

1. Define the set of subformulae of a propositional formula by recursion on the definition of propositional formulae, assuming unique readability of propositional formulae.

2. Prove soundness of the axiomatic system **H**, i.e. For every set of formulae Γ and a formula A, if $\Gamma \vdash_{\mathbf{H}} A$ then $\Gamma \models A$, by induction on the derivations in **H**.

   In that proof you will have to show that every axiom is a tautology. Prove that only for the axioms ($\rightarrow 3$) and ($\vee 3$).

3. Derive in **H**: Exercises from handouts on axiomatic systems, p.17, 1(m).

   (You may use the Deduction theorem, as well as previous exercises.)

Bonus question (challenge): Derive $\vdash_{\mathbf{H}} p \vee \neg p$.
You may use the Deduction theorem, but no other results about **H**.

DUE: March 6, 2008.